Computers and Mathematics with Applications 62 (2011) 4058–4067

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Some properties of soft topological spaces

Sabir Hussain [a,b,*], Bashir Ahmad [c]

[a] Department of Mathematics, Yanbu University, P.O. Box 31387, Yanbu Alsinaiyah, Saudi Arabia
[b] Department of Mathematics, Islamia University Bahawalpur, Pakistan
[c] Department of Mathematics, King Abdul Aziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 10 July 2011
Received in revised form 22 September 2011
Accepted 22 September 2011

Keywords:
Soft sets
Soft open (closed) sets
Soft interior (exterior)
Soft nbd
Soft boundary
Soft topological spaces

ABSTRACT

For dealing with uncertainty researchers introduced the concept of soft sets. Shabir and Naz (2011) [28], defined several basic notions on soft topology and studied many properties. In this paper, we continue investigating the properties of soft open (closed), soft nbd and soft closure. We also define and discuss the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces.

1. Introduction

Several theories, such as the theory of fuzzy sets [1], theory of intuitionists fuzzy sets [2], theory of vague sets, theory of interval mathematics [3], theory of rough sets and theory of probability [4,5] can be considered as mathematical tools for dealing with uncertainties. These theories have inherent difficulties due to the inadequacy of the parameterization tool of the theories as pointed out by Molodtsov.

In 1999 Molodtsov [6] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences.

In [7], Molodtsov applied successfully in directions such as, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement. Maji et al. [8,9] gave the first practical application of soft sets in decision making problems. In 2003, Maji et al. [9] defined and studied several basic notions of soft set theory. In 2005, Pei and Miao [10] and Chen [11] improved the work of Maji et al. [8,9].

Many researchers have contributed towards the algebraic structure of soft set theory [12–25]. The application of soft set theory in algebraic structures was introduced by Aktas and Cagman [13]. They established the basic notions of soft groups as a generalization of the idea of fuzzy groups. Jun [16] investigated BCK/BCI-algebras and their applications in ideal theory. Feng et al. [14] worked on soft semirings, soft ideals and idealistic soft semirings. Ali et al. [26], Shabir and Ali [27] studied semigroups and soft ideals over a semigroup which characterized generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup.

Recently, in 2011, Shabir and Naz [28] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as soft open and

* Corresponding author.
E-mail addresses: sabiriub@yahoo.com (S. Hussain), drbashir9@gmail.com (B. Ahmad).

0898-1221/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2011.09.051

closed sets, soft subspace, soft closure, soft nbd of a point, soft T_i-spaces, for $i = 1, 2, 3, 4$, soft regular spaces and soft normal spaces and established their several properties.

In this paper, we continue investigating the properties of soft open (closed), soft nbd and soft closure. We also define and discuss the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces.

2. Preliminaries

Now we recall some definitions and results defined and discussed in [3,6,29,14,28].

Definition 1. Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

(1) $A \subseteq B$ and
(2) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\tilde{\subset}(G, B)$.

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

Definition 3. Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4. The complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', \not A)$ where, $F' : \not A \to P(U)$ is a mapping given by $F'(\not\alpha) = U \setminus F(\alpha)$, for all $\alpha \in A$.

Let us call F' the soft complement function of F. Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.

Definition 5. A soft set (F, A) over U is said to be a NULL soft set denoted by Φ if for all $e \in A$, $F(e) = \Phi$ (null set).

Definition 6. A soft set (F, A) over U is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A$, $F(e) = U$.

Clearly $A^c = \Phi$ and $\Phi^c = A$.

Definition 7. The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$.

The following definition of the intersection of two soft sets is given as that of a bi-intersection in [29].

Definition 8. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Let X be an initial universe set and E be the non-empty set of parameters.

Definition 9. The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \setminus (G, E)$, is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 10. Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y, E) over X for which $Y(\alpha) = Y$, for all $\alpha \in E$. In particular, (X, E) will be denoted by $\tilde{X}$.

Definition 11. Let $x \in X$, then (x, E) denotes the soft set over X for which $x(\alpha) = \{x\}$, for all $\alpha \in E$.

Definition 12. Let (F, E) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F, E) over Y denoted by (Y_F, E), is defined as follows: $F_Y(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$. In other words $(Y_F, E) = \tilde{Y} \cap (F, E)$.

Definition 13. The relative complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$ where $F' : A \to P(U)$ is a mapping given by $F'(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.

Definition 14. Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

(1) $\Phi, \tilde{X}$ belong to τ.
(2) The union of any number of soft sets in τ belongs to τ.
(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 15. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 16. Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ.

Proposition 1. *Let (X, τ, E) be a soft space over X. Then*

(1) *$\Phi, \tilde{X}$ are closed soft sets over X.*
(2) *The intersection of any number of soft closed sets is a soft closed set over X.*
(3) *The union of any two soft closed sets is a soft closed set over X.*

Definition 17. Let X be an initial universe set, E be the set of parameters and $\tau = \{\Phi, \tilde{X}\}$. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete space over X.

Definition 18. Let X be an initial universe set, E be the set of parameters and let τ be the collection of all soft sets which can be defined over X. Then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Proposition 2. *Let (X, τ, E) be a soft space over X. Then the collection $\tau_\alpha = \{F(\alpha) : (F, E) \in \tau\}$ for each $\alpha \in E$, defines a topology on X.*

Proposition 2 shows that corresponding to each parameter $\alpha \in E$, we have a topology τ_α on X. Thus a soft topology on X gives a parameterized family of topologies on X.

The following example shows that any collection of soft sets need not to be a soft topology on X, even if the collection corresponding to each parameter defines a topology on X.

Example 1. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft sets over X, defined as follows

$$F_1(e_1) = \{h_2\}, \quad F_1(e_2) = \{h_1\},$$
$$F_2(e_1) = \{h_2, h_3\}, \quad F_2(e_2) = \{h_1, h_2\},$$
$$F_3(e_1) = \{h_1, h_2\}, \quad F_3(e_2) = \{h_1, h_2\},$$
$$F_4(e_1) = \{h_2\}, \quad F_4(e_2) = \{h_1, h_3\}.$$

Then τ is not a soft topology on X because $(F_2, E) \cup (F_3, E) = (G, E)$, where $G(e_1) = X$, and $G(e_2) = \{h_1, h_2\}$ and so $(G, E) \notin \tau$. Also, $\tau_{e_1} = \{\phi, X, \{h_2\}, \{h_2, h_3\}, \{h_1, h_2\}\}$ and $\tau_{e_2} = \{\phi, X, \{h_1\}, \{h_1, h_3\}, \{h_1, h_2\}\}$ are topologies on X.

Proposition 3. *Let (X, τ_1, E) and (X, τ_2, E) be two soft topological spaces over the same universe X, then $(X, \tau_1 \cup \tau_2, E)$ is a soft topological space over X.*

Definition 19. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

Theorem 1. *Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) are soft sets over X. Then*

(1) *$\overline{\Phi} = \Phi$ and $\overline{\tilde{X}} = \tilde{X}$.*
(2) *$(F, E) \tilde{\subset} \overline{(F, E)}$.*
(3) *(F, E) is a soft closed set if and only if $(F, E) = \overline{(F, E)}$.*
(4) *$\overline{\overline{(F, E)}} = (F, E)$.*
(5) *$(F, E) \tilde{\subset} (G, E)$ implies $\overline{(F, E)} \tilde{\subset} \overline{(G, E)}$.*
(6) *$\overline{(F, E) \cup (G, E)} = \overline{(F, E)} \cup \overline{(G, E)}$.*
(7) *$\overline{(F, E) \cap (G, E)} \tilde{\subset} \overline{(F, E)} \cap \overline{(G, E)}$.*

Definition 20. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then we associate with (F, E) a soft set over X, denoted by (F, E) and defined as $\overline{F}(\alpha) = \overline{F(\alpha)}$, where $\overline{F(\alpha)}$ is the closure of $F(\alpha)$ in τ_α for each $\alpha \in E$.

Proposition 4. *Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then $(\overline{F}, E)\tilde{\subset}\overline{(F, E)}$.*

Corollary 1. *Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then $(\overline{F}, E) = \overline{(F, E)}$ if and only if $(\overline{F}, E)' \in \tau$.*

Definition 21. Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and $x \in X$. Then x is said to be a soft interior point of (G, E), if there exists a soft open set (F, E) such that $x \in (F, E)\tilde{\subset}(G, E)$.

Definition 22. Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and $x \in X$. Then (G, E) is said to be a soft neighborhood of x, if there exists a soft open set (F, E) such that $x \in (F, E)\tilde{\subset}(G, E)$.

Proposition 5. *Let (X, τ, E) be a soft topological space over X. For any soft open set (F, E) over X, (F, E) is a soft neighborhood of each point of $\bigcap_{\alpha\in E} F(\alpha)$.*

Definition 23. Let (X, τ, E) be a soft topological space over X and Y be a non-empty subset of X. Then $\tau_Y = \{(Y_F, E)|(F, E) \in \tau\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

We can easily verify that τ_Y is, in fact, a soft topology on Y.

Proposition 6. *Let (X, τ, E) be a soft topological space over X and Y be a non-empty subset of X. Then (Y, τ_{α_Y}) is a subspace of (X, τ_α) for each $\alpha \in E$.*

Definition 24. Let (X, τ, E) be a soft topological space over X and Y be a non-empty subset of X. Then $\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E). We can easily verify that τ_Y is, in fact, a soft topology on Y.

Example 2. Any soft subspace of a soft discrete topological space is a soft discrete topological space.

Example 3. Any soft subspace of a soft indiscrete topological space is a soft indiscrete topological space.

Proposition 7. *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and (F, E) be a soft open set in Y. If $\tilde{Y} \in \tau$ then $(F, E) \in \tau$.*

3. Properties of soft topological spaces

Definition 25. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$, read as x belongs to the soft set (F, E), whenever $x \in F(e)$ for all $e \in E$.

Note that for any $x \in X$, $x \notin (F, E)$, if $x \notin F(e)$ for some $e \in E$.

Remark 1. The union of two soft topologies on X may not be a soft topology on X.

Example 4. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau_1 = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$, $\tau_2 = \{\Phi, \tilde{X}, (G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E)\}$ be two soft topologies defined on X where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (G_1, E), (G_2, E), (G_3, E)$ (G_4, E) and (G_5, E) are soft sets over X, defined as follows

$$F_1(e_1) = \{h_2\}, \quad F_1(e_2) = \{h_1\},$$
$$F_2(e_1) = \{h_2, h_3\}, \quad F_2(e_2) = \{h_1, h_2\},$$
$$F_3(e_1) = \{h_1, h_2\}, \quad F_3(e_2) = X,$$
$$F_4(e_1) = \{h_1, h_2\}, \quad F_4(e_2) = \{h_1, h_3\},$$
$$F_5(e_1) = \{h_2\}, \quad F_5(e_2) = \{h_1, h_2\}$$

and

$$G_1(e_1) = \{h_2\}, \quad G_1(e_2) = \{h_1\},$$
$$G_2(e_1) = \{h_2, h_3\}, \quad G_2(e_2) = \{h_1, h_2\},$$
$$G_3(e_1) = \{h_1, h_2\}, \quad G_3(e_2) = \{h_1, h_2\},$$
$$G_4(e_1) = \{h_2\}, \quad G_4(e_2) = \{h_1, h_3\}, \quad G_5(e_1) = \{h_1, h_2\}, \quad G_5(e_2) = X.$$

Now, we define

$$\tau = \tau_1 \cup \tau_2 = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (G_1, E), (G_2, E), (G_4, E), (G_5, E)\}.$$

If we take $(F_2, E) \cap (G_1, E) = (H, E)$
then

$$H(e_1) = F_2(e_1) \cap G_1(e_1) = \{h_2, h_3\} \cap \{h_2\} = \{h_2\}$$

and

$$H(e_2) = F_2(e_2) \cap G_1(e_2) = \{h_1, h_2\} \cap \{h_1, h_3\} = \{h_2\}.$$

But $(H, E) \notin \tau$. Thus τ is not a soft topology on X.

Proposition 8. *Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and $x \in X$. If x is a soft interior point of (G, E), then x is an interior point of $G(\alpha)$ in (X, τ_α), for each $\alpha \in E$.*

Proposition 9. *Let (X, τ, E) be a soft topological space over X. Then*

(1) *each $x \in X$ has a soft neighborhood;*
(2) *if (F, E) and (G, E) are soft neighborhoods of some $x \in X$, then $(F, E) \cap (G, E)$ is also a soft neighborhood of x.*
(3) *if (F, E) is a soft neighborhood of $x \in X$ and $(F, E)\tilde{\subset}(G, E)$, then (G, E) is also a soft neighborhood of $x \in X$.*

Note 1. The following example shows that the above Proposition 8 is not true in general.

Example 5. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ where (F_1, E), (F_2, E), (F_3, E), (F_4, E) and (F_5, E) are soft sets over X, defined as follows:

$$F_1(e_1) = \{h_2\}, \qquad F_1(e_2) = \{h_1\},$$
$$F_2(e_1) = \{h_2, h_3\}, \qquad F_2(e_2) = \{h_1, h_2\},$$
$$F_3(e_1) = \{h_1, h_2\}, \qquad F_3(e_2) = X,$$
$$F_4(e_1) = \{h_1, h_2\}, \qquad F_4(e_2) = \{h_1, h_3\},$$
$$F_5(e_1) = \{h_2\}, \qquad F_5(e_2) = \{h_1, h_2\}.$$

Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. It can be easily seen that

$$\tau_{e_1} = \{\phi, X, \{h_2\}, \{h_2, h_3\}, \{h_1, h_2\}\}$$

and

$$\tau_{e_2} = \{\phi, X, \{h_1\}, \{h_1, h_3\}, \{h_1, h_2\}\}$$

are topologies on X.

Since $h_2 \in X$ is soft interior point of $(G, E) = \{\{h_2, h_3\}, \{h_1, h_2\}\}$, but h_2 is not interior point of $G(e_2) = \{h_1\}$ in τ_{e_2}.

Definition 26. Let (X, τ, E) be a soft topological space over X then soft interior of soft set (F, E) over X is denoted by $(F, E)^\circ$ and is defined as the union of all soft open sets contained in (F, E).

Thus $(F, E)^\circ$ is the largest soft open set contained in (F, E).

Example 6. Let us consider the soft topological space (X, τ, E) over X in Example 5 and $(F, E) = \{\{h_2\}, \{h_1, h_3\}\}$ be soft set of soft topological space X. Then $(F, E)^\circ = \{\{h_2\}, \{h_1\}\}$.

Theorem 2. *Let (X, τ, E) be a soft topological space over X and (F, E) and (G, E) are soft sets over X. Then*

(1) *$\Phi^\circ = \Phi$ and $\tilde{X}^\circ = \tilde{X}$.*
(2) *$(F, E)^\circ\tilde{\subset}(F, E)$.*
(3) *$((F, E)^\circ)^\circ = (F, E)$.*
(4) *(F, E) is a soft open set if and only if $(F, E)^\circ = (F, E)$.*
(5) *$(F, E)\tilde{\subset}(G, E)$ implies $(F, E)^\circ\tilde{\subset}(G, E)^\circ$.*
(6) *$(F, E)^\circ \cap (G, E)^\circ = ((F, E) \cap (G, E))^\circ$.*
(7) *$(F, E)^\circ \cup (G, E)^\circ\tilde{\subset}((F, E) \cup (G, E))^\circ$.*

Proof. (1) and (2) are obvious.

(3) Since $((F, E)^\circ)$ is soft open and $((F, E)^\circ)^\circ$ is the union of all soft open subsets in X contained in $(F, E)^\circ$, $(F, E)^\circ\tilde{\subset}((F, E)^\circ)^\circ$. But $((F, E)^\circ)^\circ\tilde{\subset}(F, E)^\circ$ by (2). Hence $((F, E)^\circ)^\circ = (F, E)$.

(4) If (F, E) is a soft open sets over X then (F, E) is itself a soft open set over X which contains (F, E). So $(F, E)^\circ$ is the largest soft open set contained in (F, E) and $(F, E) = (F, E)^\circ$.

Conversely, suppose that $(F, E) = (F, E)^\circ$. Since $(F, E)^\circ$ is a soft open set, so (F, E) is a soft open set over X.

(5) Suppose that $(F, E)\tilde{\subset}(G, E)$. Since $(F, E)^\circ\tilde{\subset}(F, E)\tilde{\subset}(G, E)$. $(F, E)^\circ$ is a soft open subset of (G, E), so by definition of $(G, E)^\circ$, $(F, E)^\circ\tilde{\subset}(G, E)^\circ$.

(6) From (5), we have $((F, E) \cap (G, E))\tilde{\subset}(F, E)$, $((F, E) \cap (G, E))\tilde{\subset}(G, E)$ implies $((F, E) \cap (G, E))^\circ\tilde{\subset}(F, E)^\circ$, $((F, E) \cap (G, E))^\circ\tilde{\subset}(G, E)^\circ$ so that $((F, E) \cap (G, E))^\circ\tilde{\subset}(F, E)^\circ \cap (G, E)^\circ$. Also, since $(F, E)^\circ\tilde{\subset}(F, E)$ and $(G, E)^\circ\tilde{\subset}(G, E)$ implies $(F, E)^\circ \cap (G, E)^\circ\tilde{\subset}((F, E) \cap (G, E))$ so that $(F, E)^\circ \cap (G, E)^\circ$ is a soft open subset of $((F, E) \cap (G, E))$. Hence $(F, E)^\circ \cap (G, E)^\circ\tilde{\subset}((F, E) \cap (G, E))^\circ$. Thus $(F, E)^\circ \cap (G, E)^\circ = ((F, E) \cap (G, E))^\circ$.

(7) Since $(F, E)\tilde{\subset}((F, E) \cup (G, E))$ and $(G, E)\tilde{\subset}((F, E) \cup (G, E))$. So by (5), $(F, E)^\circ\tilde{\subset}((F, E) \cup (G, E))^\circ$ and $(G, E)^\circ\tilde{\subset}((F, E) \cup (G, E))^\circ$. So that $(F, E)^\circ \cup (G, E)^\circ\tilde{\subset}((F, E) \cup (G, E))^\circ$, since $(F, E)^\circ \cup (G, E)^\circ$ is a soft open set. □

The following example shows that the equality does not hold in Theorem 2(7).

Example 7. Let us consider the soft topological space (X, τ, E) over X in Example 5 and $(F, E) = \{\{h_2\}, \{h_1, h_3\}\}$, $(G, E) = \{\{h_1, h_3\}, \{h_1, h_2, h_3\}\}$ are soft sets of soft topological space X. Then $(F, E)^\circ = \{\{h_2\}, \{h_1\}\}$ and $(G, E)^\circ = \Phi$. $(F, E) \cup (G, E) = \{\{h_2\}, \{h_1, h_3\}\} \cup \{\{h_1, h_3\}, \{h_1, h_2, h_3\}\} = \{\{h_1, h_2, h_3\}, \{h_1, h_2, h_3\}\} = \tilde{X}$. Now $((F, E) \cup (G, E))^\circ = (\tilde{X})^\circ = \tilde{X}$ and $(F, E)^\circ \cup (G, E)^\circ = \{\{h_2\}, \{h_1\}\} \cup \Phi = \{\{h_2\}, \{h_1\}\}$. so that $(F, E)^\circ \cup (G, E)^\circ \tilde{\subset} ((F, E) \cup (G, E))^\circ$ but $((F, E) \cup (G, E))^\circ \tilde{\nsubseteq} (F, E)^\circ \cup (G, E)^\circ$.

Theorem 3. *Let (F, E) be a soft set of soft topological space over X. Then*

(1) $((F, E)')^\circ = (\overline{(F, E)})'$.
(2) $\overline{(F, E)'} = ((F, E)^\circ)'$.
(3) $(F, E)^\circ = (\overline{(F, E)'})'$.
(4) $\overline{(F, E)'} = ((F, E)^\circ)'$.

The following example shows that the equalities hold in the above theorem.

Example 8. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)$ and (F_7, E) are soft sets over X, defined as follows:

$$F_1(e_1) = \{h_1, h_2\}, \qquad F_1(e_2) = \{h_1, h_2\},$$
$$F_2(e_1) = \{h_2\}, \qquad F_2(e_2) = \{h_1, h_3\},$$
$$F_3(e_1) = \{h_2, h_3\}, \qquad F_3(e_2) = \{h_1\},$$
$$F_4(e_1) = \{h_2\}, \qquad F_4(e_2) = \{h_1\},$$
$$F_5(e_1) = \{h_1, h_2\}, \qquad F_5(e_2) = X,$$
$$F_6(e_1) = X, \qquad F_6(e_2) = \{h_1, h_2\},$$
$$F_7(e_1) = \{h_2, h_3\}, \qquad F_7(e_2) = \{h_1, h_3\}.$$

Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X.

Clearly the soft closed sets are $\tilde{X}$, $\Phi\{\{h_3\}, \{h_3\}\}$, $\{\{h_1, h_3\}, \{h_2\}\}$, $\{\{h_1\}, \{h_2, h_3\}\}$, $\{\{h_1, h_3\}, \{h_2, h_3\}\}$, $\{\{h_3\}, \phi\}$, $\{\phi, \{h_3\}\}$ and $\{\{h_1\}, \{h_2\}\}$.

Let us take $(F, E) = \{\{h_2\}, \{h_1, h_2\}\}$ then $(F, E)' = \{\{h_1, h_3\}, \{h_3\}\}$, $((F, E)')^\circ = \Phi$, $(((F, E)')^\circ)' = \tilde{X}$, $\overline{(F, E)} = \tilde{X}$, $(\overline{(F, E)})' = \Phi$, $\overline{(F, E)'} = \{\{h_1, h_3\}, \{h_2, h_3\}\}$, $(\overline{(F, E)'})' = \{\{h_2\}, \{h_1\}\}$, $(F, E)^\circ = \{\{h_2\}, \{h_1\}\}$, $((F, E)^\circ)' = \{\{h_1, h_3\}, \{h_2, h_3\}\}$. Clearly the equalities hold in the above theorem.

Definition 27. Let (X, τ, E) be a soft topological space over X then the soft exterior of soft set (F, E) over X is denoted by $(F, E)_\circ$ and is defined as $(F, E)_\circ = ((F, E)')^\circ$.

Thus x is called a soft exterior point of (F, E) if there exists a soft open set (G, E) such that $x \in (G, E) \tilde{\subset} (F, E)'$. We observe that $(F, E)_\circ$ is the largest soft open set contained in $(F, E)'$.

Example 9. Let us consider the soft topological space (X, τ, E) over X in Example 5 and $(F, E) = \{\{h_1\}, \{h_2\}\}$ be the soft set of soft topological space X. Then $(F, E)' = \{\{h_2, h_3\}, \{h_1, h_3\}\}$ and so, $(F, E)_\circ = ((F, E)')^\circ = \{\{h_2\}, \{h_1\}\}$.

Theorem 4. *Let (F, E) be a soft set of soft topological space over X. Then*

(1) $(F, E)_\circ = ((F, E)')^\circ$.
(2) $((F, E) \cup (G, E))_\circ = (F, E)_\circ \cap (G, E)_\circ$.
(3) $(F, E)_\circ \cup (G, E)_\circ \tilde{\subset} ((F, E) \cap (G, E))_\circ$.

The following example shows that the equality does not hold in Theorem 4(3).

Example 10. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E)\}$ be two soft topologies defined on X where $(G_1, E), (G_2, E), (G_3, E)$ (G_4, E) and (G_5, E) are soft sets over X, defined as follows

$$G_1(e_1) = \{h_2\}, \qquad G_1(e_2) = \{h_1\},$$
$$G_2(e_1) = \{h_2, h_3\}, \qquad G_2(e_2) = \{h_1, h_2\},$$
$$G_3(e_1) = \{h_1, h_2\}, \qquad G_3(e_2) = \{h_1, h_2\},$$
$$G_4(e_1) = \{h_2\}, \qquad G_4(e_2) = \{h_1, h_3\}, \qquad G_5(e_1) = \{h_1, h_2\}, \qquad G_5(e_2) = X.$$

Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X.

Let us take $(F, E) = \{\{h_1, h_2\}, \{h_2\}\}$, $(G, E) = \{\{h_3\}, \phi\}$.
Now $(F, E)_\circ = ((F, E)')_\circ = \{\{h_3\}, \{h_1, h_2\}\}^\circ = \phi$

and

$$(G,E)_\circ = ((G,E)')^\circ = \{\{h_1,h_2\},X\}^\circ = \{\{h_1,h_2\},\{h_1,h_2,h_3\}\}$$
$$(F,E)_\circ \cup (G,E)_\circ = \{\{h_1,h_2\},\{h_1,h_2,h_3\}\}.$$

Also

$$((F,E)\cap(G,E))_\circ = (((F,E)\cap(G,E))')^\circ = ((\{\{h_1,h_2\},\{h_2\}\}\cap\{\{h_3\},\phi\})')^\circ = ((\phi)')^\circ = (\tilde{X})^\circ = \tilde{X}.$$

Definition 28. Let (X,τ,E) be a soft topological space over X then the soft boundary of soft set (F,E) over X is denoted by $\underline{(F,E)}$ and is defined as $\underline{(F,E)} = \overline{(F,E)} \cap \overline{((F,E)')}$. Obviously $\underline{(F,E)}$ is a smallest soft closed set over X containing (F,E).

Remark 2. From the above definition it follows directly that the soft sets (F,E) and $(F,E)'$ have same soft boundary.

Example 11. In the above Example 10, let us take $(F,E) = \{\{h_2\},\{h_1,h_2\}\}$ then $\overline{(F,E)} = \tilde{X}$, $(F,E)' = \{\{h_1,h_3\},\{h_3\}\}$ and $\overline{(F,E)'} = \{\{h_1,h_3\},\{h_2,h_3\}\}$. Thus the soft boundary of (F,E) is $\underline{(F,E)} = \overline{(F,E)} \cap \overline{((F,E)')} = \tilde{X} \cap \{\{h_1,h_3\},\{h_2,h_3\}\} = \{\{h_1,h_3\},\{h_2,h_3\}\}$.

Theorem 5. *Let (F,E) be a soft set of soft topological space over X. Then the following hold:*

(1) $(\underline{(F,E)})' = (F,E)^\circ \cup ((F,E)')^\circ = (F,E)^\circ \cup (F,E)_\circ$

(2) $\overline{(F,E)} = (F,E)^\circ \cup \underline{(F,E)}$

(3) $\underline{(F,E)} = \overline{(F,E)} \cap \overline{(F,E)'} = \overline{(F,E)} - (F,E)^\circ$

(4) $(F,E)^\circ = (F,E) \setminus \underline{(F,E)}$.

Proof.

$$\begin{aligned}(1)\ (F,E)^\circ \cup ((F,E)')^\circ &= (((F,E)^\circ)')' \cup ((((F,E)')^\circ)')' \\ &= [((F,E)^\circ)' \cap ((F,E)')^\circ]' \\ &= [\overline{(F,E)'} \cap \overline{(F,E)}]' \\ &= (\underline{(F,E)})'.\end{aligned}$$

$$\begin{aligned}(2)\ (F,E)^\circ \cup \underline{(F,E)} &= (F,E)^\circ \cup (\overline{(F,E)} \cap \overline{(F,E)'}) \\ &= [(F,E)^\circ \cup \overline{(F,E)}] \cap [(F,E)^\circ \cup \overline{(F,E)'}] \\ &= \overline{(F,E)} \cap [(F,E)^\circ \cup ((F,E)^\circ)'] \\ &= \overline{(F,E)} \cap ((F,E)^\circ \cup ((F,E)^\circ)') \\ &= \overline{(F,E)} \cap \tilde{X} \\ &= \overline{(F,E)}.\end{aligned}$$

$$\begin{aligned}(3)\ \underline{(F,E)} &= \overline{(F,E)} - (F,E)^\circ \\ &= \overline{(F,E)} \cap ((F,E)^\circ)' \\ &= \overline{(F,E)} \cap \overline{(F,E)'} \quad \text{(by Theorem 3(2))}.\end{aligned}$$

$$\begin{aligned}(4)\ (F,E) \setminus \underline{(F,E)} &= (F,E) \cap \underline{(F,E)}' \\ &= (F,E) \cap ((F,E)^\circ \cup ((F,E)')^\circ) \quad \text{(by (1))} \\ &= [(F,E) \cap (F,E)^\circ] \cup [(F,E) \cap ((F,E)')^\circ] \\ &= (F,E)^\circ \cup \Phi \\ &= (F,E)^\circ. \quad \square\end{aligned}$$

Remark 3. By (3) of above Theorem 5, it is clear that $\underline{(F,E)} = \underline{(F,E)'}$.

Theorem 6. *Let (F,E) be a soft set of soft topological space over X. Then:*

(1) *(F,E) is a soft open set over X if and only if $(F,E) \cap \underline{(F,E)} = \Phi$.*

(2) *(F,E) is a soft closed set over X if and only if $\underline{(F,E)} \tilde{\subset} (F,E)$.*

Proof. (1) Let (F,E) be a soft open set over X. Then $(F,E)^\circ = (F,E)$ implies

$$\begin{aligned}(F,E) \cap \underline{(F,E)} &= (F,E)^\circ \cap \underline{(F,E)} \\ &= \Phi.\end{aligned}$$

Conversely, let $(F,E) \cap \underline{(F,E)} = \Phi$. Then $(F,E) \cap \overline{(F,E)} \cap \overline{(F,E)'} = \Phi$ or $(F,E) \cap \overline{(F,E)'} = \Phi$ or $\overline{(F,E)'} \tilde{\subset} (F,E)'$, which implies $(F,E)'$ is a soft closed set and hence (F,E) is a soft open set.

(2) Let (E, E) be a soft closed set over X. Then $\overline{(F, E)} = (F, E)$. Now $\underline{(F, E)} = \overline{(F, E)} \cap \overline{(F, E)'} \tilde{\subset} \overline{(F, E)} = (F, E)$. That is, $\underline{(F, E)} \tilde{\subset} (F, E)$.

Conversely, $\underline{(F, E)} \tilde{\subset} (F, E)$. Then $\underline{(F, E)} \cap (F, E)' = \Phi$. Since $\underline{(F, E)} = \underline{(F, E)'} = \Phi$, we have $\underline{(F, E)'} \cap (F, E)' = \Phi$. By (1), $(F, E)'$ is soft open and hence (F, E) is soft closed. $\square$

Theorem 7. *Let (F, E) be a soft set of soft topological space over X. Then the following hold:*

(1) $\underline{(F, E)} \cap (F, E)^\circ = \Phi$

(2) $\overline{(F, E)^\circ} = (F, E) \setminus \underline{(F, E)}$.

Proof. (1) follows from Theorem 5(3) and (2) follows directly by the definition of the soft boundary. $\square$

Theorem 8. *Let (F, A) and (G, B) be soft sets of soft topological space over X. Then the following hold:*

(1) $\underline{((F, A) \cup (G, B))} \tilde{\subset} [\underline{(F, A) \cap ((G, B)')}] \cup [\underline{(G, B)} \cap \overline{((F, A)')}]$

(2) $\underline{[(F, A) \cap (G, B)]} \tilde{\subset} [\underline{(F, A)} \cap \overline{(G, B)}] \cup [\underline{(G, B)} \cap \overline{(F, A)}]$.

Proof.

$$\begin{aligned}
(1)\ \underline{((F, A) \cup (G, B))} &= \overline{((F, A) \cup (G, B))} \cap \overline{(((F, A) \cup (G, B))')} \\
&= (\overline{(F, A)} \cup \overline{(G, B)}) \cap \overline{((F, A)' \cap (G, B)')} \\
&\tilde{\subset} (\overline{(F, A)} \cup \overline{(G, B)}) \cap [\overline{(F, A)'} \cap \overline{(G, B)'}] \\
&= (\overline{(F, A)} \cap \overline{(F, A)'}) \cap (\overline{(G, B)'} \cup \overline{(G, B)}) \cap [\overline{(F, A)'} \cap \overline{(G, B)'}] \\
&= (\underline{(F, A)} \cap \overline{(G, B)'}) \cup (\underline{(G, B)} \cap \overline{(F, A)'}) \\
&\tilde{\subset} \underline{(F, A)} \cup \underline{(G, B)}. \\
(2)\ \underline{[(F, A) \cap (G, B)]} &= \overline{((F, A) \cap (G, B))} \cap \overline{((F, A) \cap (G, B))'} \\
&\tilde{\subset} [\overline{(F, A)} \cap \overline{(G, B)}] \cap [\overline{((F, A)' \cup (G, B)')}] \\
&= [\overline{(F, A)} \cap \overline{(G, B)}] \cap [\overline{(F, A)'} \cup \overline{(G, B)'}] \\
&= [(\overline{(F, A)} \cap \overline{(G, B)}) \cap \overline{(F, A)'}] \cup [(\overline{(F, A)} \cap \overline{(G, B)}) \cap \overline{(G, B)'}] \\
&= (\underline{(F, A)} \cap \overline{(G, B)}) \cup (\overline{(F, A)} \cap \underline{(G, B)}). \quad \square
\end{aligned}$$

Theorem 9. *Let (F, E) be a soft set of soft topological space over X. Then the following holds:*

$$\underline{(\underline{(\underline{(F, E)})})} = \underline{(\underline{(F, E)})}.$$

Proof.

$$\begin{aligned}
\underline{(\underline{(\underline{(F, E)})})} &= \overline{(\underline{(\underline{(F, E)})})} \cap \overline{((\underline{(\underline{(F, E)})})')} \\
&= (\underline{(\underline{(F, E)})}) \cap \overline{((\underline{(\underline{(F, E)})})')}.
\end{aligned} \tag{1}$$

Now consider

$$\begin{aligned}
((\underline{(\underline{(F, E)})})') &= [\overline{(\underline{(F, E)})} \cap ((\underline{(F, E)})')]' \\
&= [\underline{(F, E)} \cap \overline{(\underline{(F, E)})'}]' \\
&= (\underline{(F, E)})' \cup (\overline{(\underline{(F, E)})'})'.
\end{aligned}$$

Therefore

$$\begin{aligned}
\overline{((\underline{(\underline{(F, E)})})')} &= \overline{[(\overline{(\underline{(F, E)})})' \cup ((\overline{(\underline{(F, E)})'}))']} \\
&= \overline{((\overline{(\underline{(F, E)})})')} \cup \overline{((\overline{(\underline{(F, E)})'}))'} \\
&= (G, E) \cup ((\overline{(\underline{(G, E)})'}))') = \tilde{X}
\end{aligned} \tag{2}$$

where $(G, E) = \overline{((\overline{(\underline{(F, E)})})')}$. From (1) and (2), we have

$$\underline{(\underline{(\underline{(F, E)})})} = \underline{(\underline{(F, E)})} \cap \tilde{X} = \underline{(\underline{(F, E)})}. \quad \square$$

Theorem 10. *Let (F,E) and (G,E) be soft open sets of soft topological space over X. Then the following hold:*

(1) $((F,E)\setminus(G,E))^\circ\tilde{\subset}(F,E)^\circ\setminus(G,E)^\circ$

(2) $\underline{(F,E)^\circ}\tilde{\subset}\underline{(F,E)}$.

Proof.

$$\begin{aligned}(1)\ ((F,E)\setminus(G,E))^\circ &= ((F,E)\cap(G,E)')^\circ\\ &= (F,E)^\circ\cap((G,E)')^\circ\\ &= (F,E)^\circ\cap(\overline{(G,E)})' \quad \text{(by Theorem 3(1))}\\ &= (F,E)^\circ\setminus\overline{(G,E)}\\ &\tilde{\subset}\ (F,E)^\circ\setminus(G,E)^\circ.\end{aligned}$$

$$\begin{aligned}(2)\ \underline{(F,E)^\circ} &= \overline{(F,E)^\circ}\cap\overline{(((F,E)^\circ)')}\\ &\tilde{\subset}\ \overline{(F,E)^\circ}\cap\overline{((\overline{(F,E)'}))} \quad \text{(by Theorem 3(4))}\\ &\tilde{\subset}\ \overline{(F,E)}\cap\overline{((F,E)')} = \underline{(F,E)}. \quad \square\end{aligned}$$

Theorem 11. *Let (F,E) be a soft set of soft topological space over X. Then $\underline{(F,E)}=\Phi$ if and only if (F,E) is a soft closed set and a soft open set.*

Proof. Suppose that $\underline{(F,E)}=\Phi$.
(i) First we prove that (F,E) is a soft closed set. Consider

$$\begin{aligned}\underline{(F,E)} = \Phi &\Rightarrow \overline{(F,E)}\cap\overline{((F,E)')}=\Phi\\ &\Rightarrow \overline{(F,E)}\tilde{\subset}((\overline{((F,E)')}))' = (F,E)^\circ \quad \text{(by Theorem 3(3))}\\ &\Rightarrow \overline{(F,E)}\tilde{\subset}(F,E) \Rightarrow \overline{(F,E)} = (F,E).\end{aligned}$$

This implies that (F,E) is a soft closed set.
(ii) Using (i), we now prove that (F,E) is a soft open set.

$$\underline{(F,E)}=\Phi \Rightarrow \overline{(F,E)}\cap\overline{((F,E)')} \quad \text{or} \quad (F,E)\cap((F,E)^\circ)'=\Phi \Rightarrow (F,E)\tilde{\subset}(F,E)^\circ \Rightarrow (F,E)^\circ=(F,E).$$

This implies that (F,E) is a soft open set.
Conversely, suppose that (F,E) is soft open and soft closed set. Then

$$\begin{aligned}\underline{(F,E)} &= \overline{(F,E)}\cap\overline{((F,E)')}\\ &= \overline{(F,E)}\cap((F,E)^\circ)' \quad \text{(by Theorem 3(4))}\\ &= (F,E)\cap(F,E)'=\Phi. \quad \square\end{aligned}$$

4. Conclusion

In the present work, we have continued to study the properties of soft open (closed) soft nbd and soft closure. We also introduce soft interior, soft exterior and soft boundary and have established several interesting properties. We hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

References

[1] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[3] K. Atanassov, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 64 (1994) 159–174.
[4] Z. Pawlak, Rough sets, International Journal of Computer Science 11 (1982) 341–356.
[5] M.B. Gorzalzany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems 21 (1987) 1–17.
[6] D. Molodtsov, Soft set theory first results, Computers and Mathematics with Applications 37 (1999) 19–31.
[7] D. Molodtsov, V.Y. Leonov, D.V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (1) (2006) 8–39.
[8] P.K. Maji, R. Biswas, R. Roy, An application of soft sets in a decision making problem, Computers and Mathematics with Applications 44 (2002) 1077–1083.
[9] P.K. Maji, R. Biswas, R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555–562.
[10] D. Pie, D. Miao, From soft sets to information systems, in: Granular computing, 2005 IEEE Inter. Conf., vol. 2, pp. 617–621.
[11] D. Chen, The parametrization reduction of soft sets and its applications, Computers and Mathematics with Applications 49 (2005) 757–763.

[12] U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, Computers and Mathematics with Applications 59 (2010) 3458–3463.
[13] H. Aktas, N. Cagman, Soft sets and soft groups, Information Sciences 1 (77) (2007) 2726–2735.
[14] F. Feng, Y.B. Jun, X. Zhao, Soft semirings, Fuzzy Sets and Systems 56 (10) (2008) 2621–2628.
[15] F. Feng, Y.B. Jun, X. Liu, L. Li, An adjustable approach to fuzzy soft set based decision making, Journal of Computational and Applied Mathematics 234 (2010) 10–20.
[16] Y.B. Jun, Soft BCK/BCI-algebras, Computers and Mathematics with Applications 56 (2008) 1408–1413.
[17] Y.B. Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Information Sciences 178 (2008) 2466–2475.
[18] Y.B. Jun, C.H. Park, Applications of soft sets in Hilbert algebras, Iranian Journal of Fuzzy Systems 6 (2) (2009) 55–86.
[19] Y.B. Jun, H.S. Kim, J. Neggers, Pseudo *d*-algebras, Information Sciences 179 (2009) 1751–1759.
[20] Y.B. Jun, K.J. Lee, A. Khan, Soft ordered semigroups, Mathematical Logic Quarterly 56 (1) (2010) 42–50.
[21] Y.B. Jun, K.J. Lee, C.H. Park, Soft set theory applied to commutative ideals, in BCK-algebras, Journal of Applied Mathematics Informatics 26 (3–4) (2008) 707–720.
[22] Y.B. Jun, K.J. Lee, C.H. Park, Soft set theory applied to ideals in *d*-algebras, Computers and Mathematics with Applications 57 (2009) 367–378.
[23] Y.B. Jun, K.J. Lee, C.H. Park, Fuzzy soft set theory applied to BCK/BCI-algebras, Computers and Mathematics with Applications 59 (2010) 3180–3192.
[24] C.H. Park, Y.B. Jun, M.A. Öztürk, Soft WS-algebras, Communications of the Korean Mathematical Society 23 (3) (2008) 314–323.
[25] Q.M. Sun, Z.L. Zhang, J. Liu, Soft sets and soft modules, in: Proceedings of Rough Sets and Knowledge Technology, Third International Conference, RSKT 2008, 17–19 May, Chengdu, China, 2008, pp. 403–409.
[26] M.I. Ali, F. Feng, X.Y. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547–1553.
[27] M. Shabir, M. Irfan Ali, Soft ideals and generalized fuzzy ideals in semigroups, New Mathematics and Natural Computation 5 (2009) 599–615.
[28] M. Shabir, M. Naz, On soft topological spaces, Computers and Mathematics with Applications 61 (2011) 1786–1799.
[29] Z. Kong, L. Gao, L. Wong, S. Li, The normal parameter reduction of soft sets and its algorithm, Journal of Computational and Applied Mathematics 21 (2008) 941–945.